Chapter 7

Exponential and Logarithmic Functions

7.1 -7.2

General Equation:

Y = ca^{x}

c = initial amount

y = final amount

a = growth or decay factor If a > 1, it is growth, if a is between 0 and 1, it is decay

a = (1 + r) growth, (1 - r) decay

x = time

Exponential Growth:

y = $a(1 + r)^{t}$

a = initial amount

r = percent increase written as a decimal

1 + r = growth factor

Exponential Decay:

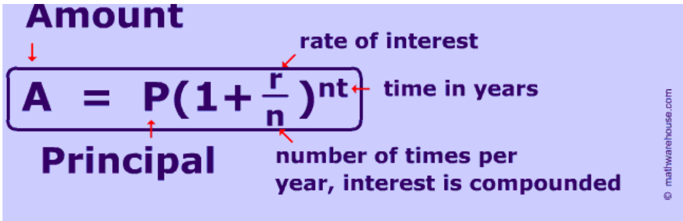
y = $a(1 - r)^{t}$

a = initial amount

r = percent decrease written as a decimal

1 - r = decay factor

Compound Interest:



A = P (1 + r/n) ^{nt}

P = Principal, initial deposit, starting amount

r = annual interest rate, written as a decimal

n = compounded n times per year (annually = 1, biannually = 2, quarterly = 4, monthly = 12)

t = time in years

A = the amount in the account after t years

<u>**Expression**</u>	<u>**Growth**</u> <u>**or**</u> <u>**decay**</u>	<u>**Growth**</u> <u>**or**</u> <u>**decay**</u> <u>**rate**</u>
$.02 (1.6)^{x}$	Growth(1 +.6)	60%
$5/4(4/5)^{x}$	Decay (1 - 4/5)	20% (1/5)
$(2.6)^{x}$	Growth(1+ 1.6)	160%
$1.5(.06)^{x}$	Decay (1 - .06)	94%

- ❏ A = 1 (+ or -) r A > 1 = growth factor, A between 0 or 1 is decay
- ❏ If it's growth, it's going to be plus
- ❏ Convert growth, to percentage
- ❏ If it's decay, subtract given % rate from 1 , (1 - r) to determine percentage of decay
- ❏ To find a graph, use calculator or plot years and values and record data, show a slope of decay (down) or growth (up)

Natural Base e :

- Natural Base e is irrational like pii
- e is approx. 2.71828
- Used in the continuous interest formula
- Use 2nd division to get e, or 2nd LN to get e

A = Pe^rt

A= Amount

P = Principal

R = rate

t = time

Ex. How much in account if $2000 invested at 9% for 15 years compounded continuously?

A = 2000e $^{(.09*15)}$ = $7714.85

Using e in exponents:

y = ae ^{rx}

y = final amount

a = initial amount

e = e

r = if + or > than 0 it's growth, if it's less than zero, then it's decay

x= time

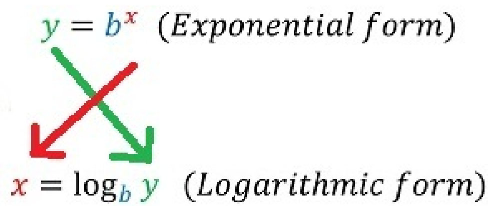
examples:

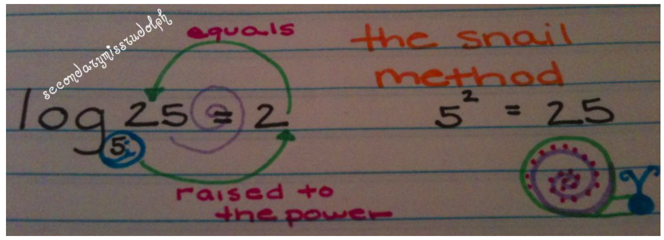
2e $^{2/6e}$ = e/3 (simplify)

e $^{x+1}$ * e $^{2x-6}$ = e $^{3x-5}$ (combine like terms)

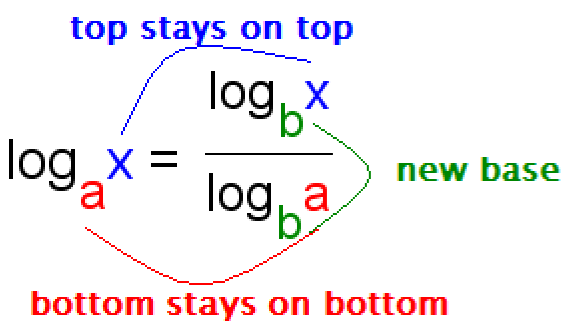
e ^{5x} / e ^{3} = e $^{5x-3}$

Logarithm of y with base b:





<u>Change</u> <u>of</u> <u>Base</u> <u>formula</u>:



Divide top by bottom.

Examples:

Solve $\log_2 4$.

$$\log_2 4 = \frac{\log_{10} 4}{\log_{10} 2} \approx \frac{0.602060}{0.301030} \approx 2$$

Check $2^2 = 4$

Solve $\log_5 22$.

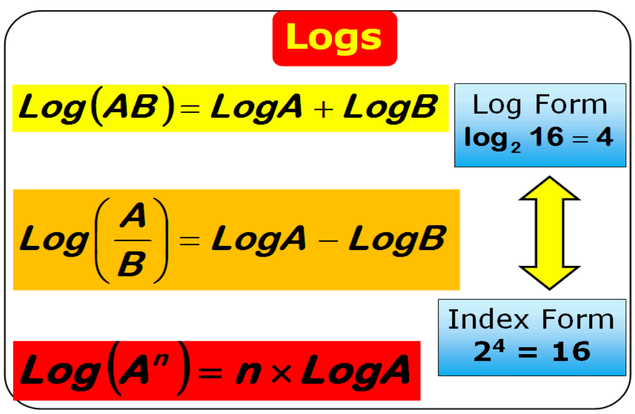
$$\log_5 22 = \frac{\ln 22}{\ln 5} \approx \frac{3.091042}{1.609438} \approx 1.920573$$

Check $5^{1.920573} \approx 22.00001 \approx 22$

Properties of Logarithms:

Property	Definition	Example
Product	$\log_b mn = \log_b m + \log_b n$	$\log_3 9x = \log_3 9 + \log_3 x$
Quotient	$\log_b \frac{m}{n} = \log_b m - \log_b n$	$\log_{\frac{1}{4}} \frac{4}{5} = \log_{\frac{1}{4}} 4 - \log_{\frac{1}{4}} 5$
Power	$\log_b m^p = p \cdot \log_b m$	$\log_2 8^x = x \cdot \log_2 8$
Equality	If $\log_b m = \log_b n$,, then *m* = *n*.	$\log_8(3x-4) = \log_8(5x+2)$ so, 3x – 4 = 5x+2

Expanding and Condensing Logarithms:



Rules for Expanding:

1. Count the number of terms (big #s and letters)

2. Rewrite the log base for as many terms there are

3. Place # in back of log, letter after log, and exponents in front of log with the corresponding letter.

4. Square roots turn to fractional exponents.

5. To undo multiplication, add, to undo division subtract.

Example:

$$\log_5(3xyk) = \log_5(3) + \log_5(x) + \log_5(y) + \log_5(k)$$

<u>Rules</u> <u>for</u> <u>condensing</u>:

1. Count the number of times Log is written, that's the number of terms you'll have.

2. Determine what process to use: Adding terms, multiply. Subtracting terms, divide.

3. Rewrite log and list all terms behind it. If there are #s in front of log, change back to exponents. Fractions can turn back to square roots or stay as fractional exponents.

*** Big Rule:**

- To undo $\sqrt{\ }$, put value to exponent of ½.
- To undo $\sqrt[3]{\ }$, put value to exponent of ⅓.
- To undo $\sqrt[n]{\ }$, put value to exponent of 1/n.
- a number raised to the -1, is written as a fraction. Ex 6^{-1} = ⅙

<u>Solving Exponential and Logarithmic Equations</u>

* Recognize what to do to get x alone, when x is in a different location.

Example:

1. $x^3 = 57$ (what number to the 3rd power = 57) Raise each side to 1/3 power to get x to the 1st power.

2. $3^x = 82$ (3 to what power = 82) $\log_3 82 = x$

3. $3^5 = x$ (3 to the 5th power = 243) $\log_3 x = 5$

Example:

$2 + 3^{x-5} = 18$ Isolate base to an exponent on one side

$3^{x-5} = 16$ Convert to log form

$\log_3 16 = x - 5$ Change of base formula

log(16)/log(3) = x - 5 Solve for x (divide, and add 5)

7.524 = x

Example:

$2 \log_4 (x-2) = 3$ Get logarithm alone, divide by 2.

$\log_4 (x-2) = 3/2$ Convert to exponential form.

$4^{3/2} = x - 2$ Solve for x.

$10 = x$

One to one property:

If $a^x = a^y$, then $x = y$

Solve $2^{x+5} = 2^{3x-1}$ Solve as linear equation because bases are the same.

$x + 5 = 3x - 1$ Becomes linear. Solve for x.

$x = 3$

If $\log_a$ of x = $\log_a$ of y

then x = y , if log bases are the same.

Example:

$\log_2 (x+4)^2 = \log_2 64$

$(x+4)^2 = 64$ Solve as a quadratic with +/-

$x + 4 = \pm 8$

x = 4, or -12 Make sure to double check, (-) solutions may not satisfy the log.

Example:

$2^{X+1} = 8^{X-4}$ Rewrite it to make bases the same. Raise 2 and the factor to the 3rd power

$2^{X+1} = 2^3$ (3x-12)

$x + 1 = 3x - 12$ Solve for x.