oefficients may differ. Examples of like terms are

$$19m^5 \quad \text{and} \quad 14m^5,$$
$$6y^9, \quad -37y^9, \quad \text{and} \quad y^9,$$
$$3pq \quad \text{and} \quad -2pq,$$
$$2xy^2 \quad \text{and} \quad -xy^2.$$

g the distributive property, we combine, or add, like terms by adding their ficients.

XAMPLE 2 Adding Like Terms

plify each expression by adding like terms.

$-4x^3 + 6x^3 = (-4 + 6)x^3 = 2x^3$ Distributive property

$9x^6 - 14x^6 + x^6 = (9 - 14 + 1)x^6 = -4x^6$

$12m^2 + 5m + 4m^2 = (12 + 4)m^2 + 5m = 16m^2 + 5m$

$3x^2y + 4x^2y - x^2y = (3 + 4 - 1)x^2y = 6x^2y$

Now Try Exercises 17, 23, and 27.

In Example 2(c), we cannot combine $16m^2$ and $5m$. These two terms are unlike use the exponents on the variables are different. *Unlike terms* have different varis or different exponents on the same variables.

ECTIVE 3 Know the vocabulary for polynomials. A **polynomial in x** is a term or sum of a finite number of terms of the form ax^n, for any real number a and any le number n. For example,

$$16x^8 - 7x^6 + 5x^4 - 3x^2 + 4$$

polynomial in x. (The 4 can be written as $4x^0$.) This polynomial is written in **deding powers** of the variable, since the exponents on x decrease from left to right. the other hand,

$$2x^3 - x^2 + \frac{4}{x}$$

ot a polynomial in x, since a variable appears in a denominator. Of course, we ld define *polynomial* using any variable and not just x, as in Example 2(c). In fact, ynomials may have terms with more than one variable, as in Example 2(d).

The **degree of a term** is the sum of the exponents on the variables. For example, has degree 4, while $6x^{17}$ has degree 17. The term $5x$ has degree 1, -7 has degree 0 ce -7 can be written as $-7x^0$), and $2x^2y$ has degree $2 + 1 = 3$. (y has an expot of 1.) The **degree of a polynomial** is the greatest degree of any nonzero term of

A polynomial with exactly two terms is called a **binomial.** (*Bi-* means "two," as in *bi*cycle.) Examples are

$$-9x^4 + 9x^3, \qquad 8m^2 + 6m, \qquad \text{and} \qquad 3m^5n^2 - 9m^2n^4.$$

A polynomial with only one term is called a **monomial.** (*Mon(o)-* means "one," as in *mono*rail.) Examples are

$$9m, \qquad -6y^5, \qquad a^2b^2, \qquad \text{and} \qquad 6.$$

EXAMPLE 3 Classifying Polynomials

For each polynomial, first simplify if possible by combining like terms. Then give the degree and tell whether it is a monomial, a binomial, a trinomial, or none of these.

(a) $2x^3 + 5$

The polynomial cannot be simplified. The degree is 3. The polynomial is a binomial.

(b) $4xy - 5xy + 2xy$

Add like terms to simplify: $4xy - 5xy + 2xy = xy$, which is a monomial of degree 2.

Now Try Exercises 29 and 31.

OBJECTIVE 4 Evaluate polynomials. A polynomial usually represents different numbers for different values of the variable, as shown in the next example.

EXAMPLE 4 Evaluating a Polynomial

Find the value of $3x^4 + 5x^3 - 4x - 4$ when $x = -2$ and when $x = 3$.

First, substitute -2 for x.

$$\begin{aligned} 3x^4 + 5x^3 - 4x - 4 &= 3(-2)^4 + 5(-2)^3 - 4(-2) - 4 \\ &= 3 \cdot 16 + 5 \cdot (-8) - 4(-2) - 4 && \text{Apply exponents.} \\ &= 48 - 40 + 8 - 4 && \text{Multiply.} \\ &= 12 && \text{Add and subtract.} \end{aligned}$$

Next, replace x with 3.

$$\begin{aligned} 3x^4 + 5x^3 - 4x - 4 &= 3(3)^4 + 5(3)^3 - 4(3) - 4 \\ &= 3 \cdot 81 + 5 \cdot 27 - 12 - 4 \\ &= 362 \end{aligned}$$

Now Try Exercise 37.

CAUTION Notice the use of parentheses around the numbers that are substituted for the variable in Example 4. This is particularly important when

Adding Polynomials

To add two polynomials, add like terms.

EXAMPLE 5 Adding Polynomials Vertically

Add $6x^3 - 4x^2 + 3$ and $-2x^3 + 7x^2 - 5$.

Write like terms in columns.

$$\begin{array}{r} 6x^3 - 4x^2 + 3 \\ \underline{-2x^3 + 7x^2 - 5} \end{array}$$

Now add, column by column.

$$\begin{array}{rrr} 6x^3 & -4x^2 & 3 \\ \underline{-2x^3} & \underline{7x^2} & \underline{-5} \\ 4x^3 & 3x^2 & -2 \end{array}$$

Add the three sums together.

$$4x^3 + 3x^2 + (-2) = 4x^3 + 3x^2 - 2$$

Now Try Exercise 45.

The polynomials in Example 5 also can be added horizontally.

EXAMPLE 6 Adding Polynomials Horizontally

Add $6x^3 - 4x^2 + 3$ and $-2x^3 + 7x^2 - 5$.

Write the sum as

$$(6x^3 - 4x^2 + 3) + (-2x^3 + 7x^2 - 5).$$

Use the associative and commutative properties to rewrite this sum with the parentheses removed and with the subtractions changed to additions of inverses.

$$6x^3 + (-4x^2) + 3 + (-2x^3) + 7x^2 + (-5)$$

Place like terms together.

$$6x^3 + (-2x^3) + (-4x^2) + 7x^2 + 3 + (-5)$$

Combine like terms to get

$$4x^3 + 3x^2 + (-2), \quad \text{or} \quad 4x^3 + 3x^2 - 2,$$

the same answer found in Example 5.

Now Try Exercise 61.

Earlier, we defined the difference $x - y$ as $x + (-y)$. (We find the difference $x - y$ by adding x and the opposite of y.) For example,

$$7 - 2 = 7 + (-2) = 5 \quad \text{and} \quad -8 - (-2) = -8 + 2 = -6.$$

A similar method is used to subtract polynomials.

Subtracting Polynomials

To subtract two polynomials, change all the signs in the second polynomial and add the result to the first polynomial.

EXAMPLE 7 Subtracting Polynomials

(a) Perform the subtraction $(5x - 2) - (3x - 8)$.

By the definition of subtraction,

$$(5x - 2) - (3x - 8) = (5x - 2) + [-(3x - 8)].$$

As shown in Chapter 1, the distributive property gives

$$-(3x - 8) = -1(3x - 8) = -3x + 8,$$

so

$$\begin{aligned} (5x - 2) - (3x - 8) &= (5x - 2) + (-3x + 8) \\ &= 2x + 6. \end{aligned}$$

(b) Subtract $6x^3 - 4x^2 + 2$ from $11x^3 + 2x^2 - 8$.

Write the problem.

$$(11x^3 + 2x^2 - 8) - (6x^3 - 4x^2 + 2)$$

Change all signs in the second polynomial and add.

$$(11x^3 + 2x^2 - 8) + (-6x^3 + 4x^2 - 2) = 5x^3 + 6x^2 - 10$$

To check a subtraction problem, use the fact that if $a - b = c$, then $a = b + c$. For example, $6 - 2 = 4$, so check by writing $6 = 2 + 4$, which is correct. Check the polynomial subtraction above by adding $6x^3 - 4x^2 + 2$ and $5x^3 + 6x^2 - 10$. Since the sum is $11x^3 + 2x^2 - 8$, the subtraction was performed correctly.

Now Try Exercise 59.

Subtraction also can be done in columns (vertically). We use vertical subtraction in Section 5.7 when we divide polynomials.

EXAMPLE 8 Subtracting Polynomials Vertically

Use the method of subtracting by columns to find

$$(14y^3 - 6y^2 + 2y - 5) - (2y^3 - 7y^2 - 4y + 6).$$

Arrange like terms in columns.

$$\begin{array}{r} 14y^3 - 6y^2 + 2y - 5 \\ \underline{2y^3 - 7y^2 - 4y + 6} \end{array}$$

Change all signs in the second row, and then add.

$$\begin{array}{rl} 14y^3 - 6y^2 + 2y - 5 & \\ \underline{-2y^3 + 7y^2 + 4y - 6} & \text{Change all signs.} \\ 12y^3 + y^2 + 6y - 11 & \text{Add.} \end{array}$$

Now Try Exercise 55.

Either the horizontal or the vertical method may be used to add and subtract polynomials.

We add and subtract polynomials in more than one variable by combining like terms, just as with single variable polynomials.

EXAMPLE 9 Adding and Subtracting Polynomials with More Than One Variable

Add or subtract as indicated.

(a) $(4a + 2ab - b) + (3a - ab + b) = 4a + 2ab - b + 3a - ab + b$

$= 7a + ab$ Combine like terms.

(b) $(2x^2y + 3xy + y^2) - (3x^2y - xy - 2y^2)$

$= 2x^2y + 3xy + y^2 - 3x^2y + xy + 2y^2$

$= -x^2y + 4xy + 3y^2$

Now Try Exercises 73 and 75.

OBJECTIVE 6 Graph equations defined by polynomials of degree 2. In Chapter 3 we introduced graphs of straight lines. These graphs were defined by linear equations (which are actually polynomial equations of degree 1). By selective point-plotting, we can graph polynomial equations of degree 2.

EXAMPLE 10 Graphing Equations Defined by Polynomials with Degree 2

Graph each equation.

(a) $y = x^2$

Select several values for x; then find the corresponding y-values. For example, selecting $x = 2$ gives

$$y = 2^2 = 4,$$

and so the point (2, 4) is on the graph of $y = x^2$. (Recall that in an ordered pair such as (2, 4), the x-value comes first and the y-value second.) We show some ordered pairs that satisfy $y = x^2$ in the table next to Figure 1. If we plot the ordered pairs from the table on a coordinate system and draw a smooth curve through them, we obtain the graph shown in Figure 1.

x	y
3	9
2	4
1	1
0	0
−1	1
−2	4
−3	9

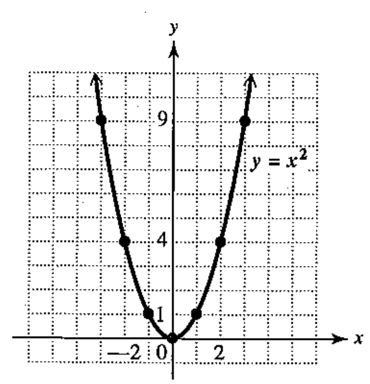


FIGURE 1

The graph of $y = x^2$ is the graph of a function, since each input x is related to just one output y. The curve in Figure 1 is called a **parabola.** The point (0, 0), the lowest point on this graph, is called the **vertex** of the parabola. The vertical line through the vertex (the y-axis here) is called the **axis** of the parabola. The axis of a parabola is a **line of symmetry** for the graph. If the graph is folded on this line, the two halves will match.

(b) $y = -x^2 + 3$

Once again plot points to obtain the graph. For example, if $x = -2$,

$$y = -(-2)^2 + 3 = -4 + 3 = -1.$$

This point and several others are shown in the table that accompanies the graph in Figure 2. The vertex of this parabola is (0, 3). This time the vertex is the *highest* point on the graph. The graph opens downward because x^2 has a negative coefficient.

x	y
−2	−1
−1	2
0	3
1	2
2	−1

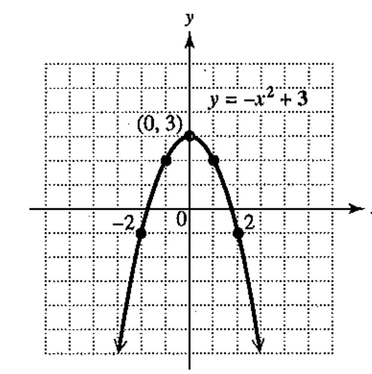


FIGURE 2

Now Try Exercises 87 and 91.

NOTE All polynomials of degree 2 have parabolas as their graphs. When graphing by plotting points, it is necessary to continue finding points until the vertex and points on either side of it are located. (In this section, all parabolas have their vertices on the x-axis or the y-axis.)

CONNECTIONS

In Section 3.6 we introduced the idea of a function: for every input x, there is one output y. For example, according to the U.S. National Aeronautics and Space Administration (NASA), the budget in millions of dollars for space station research for 1996–2001 can be approximated by the polynomial equation

$$y = -10.25x^2 - 126.04x + 5730.21,$$

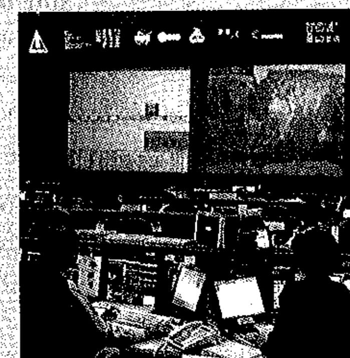
where $x = 0$ represents 1996, $x = 1$ represents 1997, and so on, up to $x = 5$ representing 2001. The actual budget for 1998 was 5327 million dollars; an input of $x = 2$ (for 1998) in the equation gives approximately $y = 5437$. Considering the magnitude of the numbers, this is a good approximation.

(continued)

For Discussion or Writing

Use the given polynomial equation to approximate the budget in other years between 1996 and 2001. Compare to the actual figures given here.

Year	Budget (in millions of dollars)
1996	5710
1997	5675
1998	5327
1999	5306
2000	5077
2001	4832



5.4 EXERCISES

For Extra Help

- Student's Solutions Manual
- MyMathLab
- InterAct Math Tutorial Software
- AW Math Tutor Center
- MathXL
- Digital Video Tutor CD 10/Videotape 8

Fill in each blank with the correct response.

1. In the term $7x^5$, the coefficient is ____ and the exponent is ____.

2. The expression $5x^3 - 4x^2$ has ________ term(s). (how many?)

3. The degree of the term $-4x^8$ is ____.

4. The polynomial $4x^2 - y^2$ ________ an example of a trinomial. (is/is not)

5. When $x^2 + 10$ is evaluated for $x = 4$, the result is ____.

6. $5x$— $+ 3x^3 - 7x$ is a trinomial of degree 4.

7. $3xy + 2xy - 5xy =$ ____.

8. ____ is an example of a monomial with coefficient 5, in the variable x, having degree 9.

For each polynomial, determine the number of terms and name the coefficients of the terms. See Example 1.

9. $6x^4$ **10.** $-9y^5$ **11.** t^4 **12.** s^7

13. $-19r^2 - r$ **14.** $2y^3 - y$ **15.** $x + 8x^2 + 5x^3$ **16.** $v - 2v^3 - v^7$

In each polynomial, add like terms whenever possible. Write the result in descending powers of the variable. See Example 2.

17. $-3m^5 + 5m^5$ **18.** $-4y^3 + 3y^3$

19. $2r^5 + (-3r^5)$ **20.** $-19y^2 + 9y^2$

21. $.2m^5 - .5m^2$ **22.** $-.9y + .9y^2$

23. $-3x^5 + 2x^5 - 4x^5$ **24.** $6x^3 - 8x^3 + 9x^3$

25. $-4p^7 + 8p^7 + 5p^9$ **26.** $-3a^8 + 4a^8 - 3a^2$

27. $-4xy^2 + 3xy^2 - 2xy^2 + xy^2$ **28.** $3pr^5 - 8pr^5 + pr^5 + 2pr^5$

For each polynomial, first simplify, if possible, and write it in descending powers of the variable. Then give the degree of the resulting polynomial and tell whether it is a monomial, binomial, trinomial, *or* none of these. *See Example 3.*

29. $6x^4 - 9x$ **30.** $7t^3 - 3t$

31. $5m^4 - 3m^2 + 6m^4 - 7m^3$ **32.** $6p^5 + 4p^3 - 8p^5 + 10p^2$

33. $\frac{5}{3}x^4 - \frac{2}{3}x^4$ **34.** $\frac{4}{5}r^6 + \frac{1}{5}r^6$

35. $.8x^4 - .3x^4 - .5x^4 + 7$ **36.** $1.2t^3 - .9t^3 - .3t^3 + 9$

Find the value of each polynomial when **(a)** $x = 2$ *and when* **(b)** $x = -1$. *See Example 4.*

37. $2x^5 - 4x^4 + 5x^3 - x^2$ **38.** $2x^2 + 5x + 1$

39. $-3x^2 + 14x - 2$ **40.** $-2x^2 + 3$

41. $2x^2 - 3x - 5$ **42.** $x^2 + 5x - 10$

Add. See Example 5.

43. $\begin{array}{r} 2x^2 - 4x \\ \underline{3x^2 + 2x} \end{array}$ **44.** $\begin{array}{r} -5y^3 + 3y \\ \underline{8y^3 - 4y} \end{array}$ **45.** $\begin{array}{r} 3m^2 + 5m + 6 \\ \underline{2m^2 - 2m - 4} \end{array}$

46. $\begin{array}{r} 4a^3 - 4a^2 - 4 \\ \underline{6a^3 + 5a^2 - 8} \end{array}$ **47.** $\begin{array}{r} \frac{2}{3}x^2 + \frac{1}{5}x + \frac{1}{6} \\ \underline{\frac{1}{2}x^2 - \frac{1}{3}x + \frac{2}{3}} \end{array}$ **48.** $\begin{array}{r} \frac{4}{7}y^2 - \frac{1}{5}y + \frac{7}{9} \\ \underline{\frac{1}{3}y^2 - \frac{1}{3}y + \frac{2}{5}} \end{array}$

49. $\begin{array}{r} 9m^3 - 5m^2 + 4m - 8 \\ \underline{-3m^3 + 6m^2 + 8m - 6} \end{array}$ **50.** $\begin{array}{r} 12r^5 + 11r^4 - 7r^3 - 2r^2 \\ \underline{-8r^5 + 10r^4 + 3r^3 + 2r^2} \end{array}$

Subtract. See Example 8.

51. $\begin{array}{r} 5y^3 - 3y^2 \\ \underline{2y^3 + 8y^2} \end{array}$ **52.** $\begin{array}{r} -6t^3 + 4t^2 \\ \underline{8t^3 - 6t^2} \end{array}$

53. $\begin{array}{r} 12x^4 - x^2 + x \\ \underline{8x^4 + 3x^2 - 3x} \end{array}$ **54.** $\begin{array}{r} 13y^5 - y^3 - 8y^2 \\ \underline{7y^5 + 5y^3 + y^2} \end{array}$

55. $\begin{array}{r} 12m^3 - 8m^2 + 6m + 7 \\ \underline{-3m^3 + 5m^2 - 2m - 4} \end{array}$ **56.** $\begin{array}{r} 5a^4 - 3a^3 + 2a^2 - a + 6 \\ \underline{-6a^4 + a^3 - a^2 + a - 1} \end{array}$

57. After reading Examples 5–8, do you have a preference regarding horizontal or vertical addition and subtraction of polynomials? Explain your answer.

58. Write a paragraph explaining how to add and subtract polynomials. Give an example using addition.

Perform each indicated operation. See Examples 6 and 7.

59. $(8m^2 - 7m) - (3m^2 + 7m - 6)$ **60.** $(x^2 + x) - (3x^2 + 2x - 1)$

61. $(16x^3 - x^2 + 3x) + (-12x^3 + 3x^2 + 2x)$ **62.** $(-2b^6 + 3b^4 - b^2) + (b^6 + 2b^4 + 2b^2)$

63. $(7y^4 + 3y^2 + 2y) - (18y^4 - 5y^2 + y)$ **64.** $(8t^5 + 3t^3 + 5t) - (19t^5 - 6t^3 + t)$

1997, and so on. If we evaluate the polynomial for a specific input value x, we will get one and only one output value y as a result. This idea is basic to the study of functions, one of the most important concepts in mathematics. ***Work Exercises 97–100 in order.***

97. If gasoline costs \$1.25 per gallon, then the monomial $1.25x$ gives the cost of x gallons. Evaluate this monomial for 4, and then use the result to fill in the blanks: If ____ gallons are purchased, the cost is ____.

98. If it costs \$15 to rent a chain saw plus \$2 per day, the binomial $2x + 15$ gives the cost to rent the chain saw for x days. Evaluate this polynomial for 6 and then use the result to fill in the blanks: If the saw is rented for ____ days, the cost is ____.

99. If an object is thrown upward under certain conditions, its height in feet is given by the trinomial $-16x^2 + 60x + 80$, where x is in seconds. Evaluate this polynomial for 2.5 and then use the result to fill in the blanks: If ____ seconds have elapsed, the height of the object is ____ feet.

100. The polynomial $2.69x^2 + 4.75x + 452.43$ gives a good approximation for the number of revenue passenger miles, in billions, for the U.S. airline industry during the period from 1990 to 1995, where $x = 0$ represents 1990. Use this polynomial to approximate the number of revenue passenger miles in 1991. (*Hint:* Any power of 1 is equal to 1, so simply add the coefficients and the constant.) (*Source:* Air Transportation Association of America.)

5.5 Multiplying Polynomials

OBJECTIVES

1. Multiply a monomial and a polynomial.
2. Multiply two polynomials.
3. Multiply binomials by the FOIL method.

OBJECTIVE 1 Multiply a monomial and a polynomial. As shown earlier, we find the product of two monomials by using the rules for exponents and the commutative and associative properties. For example,

$$(-8m^6)(-9n^6) = (-8)(-9)(m^6)(n^6) = 72m^6n^6.$$

CAUTION Do not confuse *addition* of terms with *multiplication* of terms. For example,

$$7q^5 + 2q^5 = 9q^5, \quad \text{but} \quad (7q^5)(2q^5) = 7 \cdot 2q^{5+5} = 14q^{10}.$$

To find the product of a monomial and a polynomial with more than one term, we use the distributive property and multiplication of monomials.

EXAMPLE 1 Multiplying Monomials and Polynomials

Use the distributive property to find each product.

(a) $4x^2(3x + 5)$

$$4x^2(3x + 5) = 4x^2(3x) + 4x^2(5) \quad \text{Distributive property}$$
$$= 12x^3 + 20x^2 \quad \text{Multiply monomials.}$$

(b) $-8m^3(4m^3 + 3m^2 + 2m - 1)$

$$= -8m^3(4m^3) + (-8m^3)(3m^2) + (-8m^3)(2m) + (-8m^3)(-1) \quad \text{Distributive property}$$
$$= -32m^6 - 24m^5 - 16m^4 + 8m^3 \quad \text{Multiply monomials.}$$

Now Try Exercises 11 and 19.

OBJECTIVE 2 Multiply two polynomials. We use the distributive property repeatedly to find the product of any two polynomials. For example, to find the product of the polynomials $x^2 + 3x + 5$ and $x - 4$, think of $x - 4$ as a single quantity and use the distributive property as follows.

$$(x^2 + 3x + 5)(x - 4) = x^2(x - 4) + 3x(x - 4) + 5(x - 4)$$

Now use the distributive property three times to find $x^2(x - 4)$, $3x(x - 4)$, and $5(x - 4)$.

$$x^2(x - 4) + 3x(x - 4) + 5(x - 4)$$
$$= x^2(x) + x^2(-4) + 3x(x) + 3x(-4) + 5(x) + 5(-4)$$
$$= x^3 - 4x^2 + 3x^2 - 12x + 5x - 20 \quad \text{Multiply monomials.}$$
$$= x^3 - x^2 - 7x - 20 \quad \text{Combine like terms.}$$

This example suggests the following rule.

Multiplying Polynomials

To multiply two polynomials, multiply each term of the second polynomial by each term of the first polynomial and add the products.

EXAMPLE 2 Multiplying Two Polynomials

Multiply $(m^2 + 5)(4m^3 - 2m^2 + 4m)$.

Multiply each term of the second polynomial by each term of the first.

$$(m^2 + 5)(4m^3 - 2m^2 + 4m)$$
$$= m^2(4m^3) + m^2(-2m^2) + m^2(4m) + 5(4m^3) + 5(-2m^2) + 5(4m)$$
$$= 4m^5 - 2m^4 + 4m^3 + 20m^3 - 10m^2 + 20m$$
$$= 4m^5 - 2m^4 + 24m^3 - 10m^2 + 20m \quad \text{Combine like terms.}$$

Now Try Exercise 25.

When at least one of the factors in a product of polynomials has three or more terms, the multiplication can be simplified by writing one polynomial above the other vertically.

EXAMPLE 3 Multiplying Polynomials Vertically

Multiply $(x^3 + 2x^2 + 4x + 1)(3x + 5)$ using the vertical method.

Write the polynomials as follows.

$$\begin{array}{r} x^3 + 2x^2 + 4x + 1 \\ 3x + 5 \\ \hline \end{array}$$

It is not necessary to line up terms in columns, because any terms may be multiplied (not just like terms). Begin by multiplying each of the terms in the top row by 5.

$$\begin{array}{rl} x^3 + 2x^2 + 4x + 1 & \\ 3x + 5 & \\ \hline 5x^3 + 10x^2 + 20x + 5 & \quad 5(x^3 + 2x^2 + 4x + 1) \end{array}$$

Notice how this process is similar to multiplication of whole numbers. Now multiply each term in the top row by $3x$. Be careful to place like terms in columns, since the final step will involve addition (as in multiplying two whole numbers).

$$\begin{array}{rl} x^3 + 2x^2 + 4x + 1 & \\ 3x + 5 & \\ \hline 5x^3 + 10x^2 + 20x + 5 & \\ 3x^4 + 6x^3 + 12x^2 + 3x \qquad\qquad & \quad 3x(x^3 + 2x^2 + 4x + 1) \end{array}$$

Add like terms.

$$\begin{array}{r} x^3 + 2x^2 + 4x + 1 \\ 3x + 5 \\ \hline 5x^3 + 10x^2 + 20x + 5 \\ 3x^4 + 6x^3 + 12x^2 + 3x \qquad \\ \hline 3x^4 + 11x^3 + 22x^2 + 23x + 5 \end{array}$$

The product is $3x^4 + 11x^3 + 22x^2 + 23x + 5$.

Now Try Exercise 29.

EXAMPLE 4 Multiplying Polynomials with Fractional Coefficients Vertically

Find the product of $4m^3 - 2m^2 + 4m$ and $\frac{1}{2}m^2 + \frac{5}{2}$.

$$\begin{array}{rl} 4m^3 - 2m^2 + 4m & \\ \frac{1}{2}m^2 + \frac{5}{2} & \\ \hline 10m^3 - 5m^2 + 10m & \quad \text{Terms of top row multiplied by } \tfrac{5}{2} \\ 2m^5 - m^4 + 2m^3 \qquad\qquad & \quad \text{Terms of top row multiplied by } \tfrac{1}{2}m^2 \\ \hline 2m^5 - m^4 + 12m^3 - 5m^2 + 10m & \quad \text{Add.} \end{array}$$

Now Try Exercise 35.

We can use a rectangle to model polynomial multiplication. For example, to find the product

$$(2x + 1)(3x + 2),$$

label a rectangle with each term as shown here.

	$3x$	2
$2x$		
1		

Now put the product of each pair of monomials in the appropriate box.

	$3x$	2
$2x$	$6x^2$	$4x$
1	$3x$	2

The product of the original binomials is the sum of these four monomial products.

$$\begin{aligned}(2x + 1)(3x + 2) &= 6x^2 + 4x + 3x + 2 \\ &= 6x^2 + 7x + 2\end{aligned}$$

This approach can be extended to polynomials with any number of terms.

OBJECTIVE 3 Multiply binomials by the FOIL method. In algebra, many of the polynomials to be multiplied are both binomials (with just two terms). For these products, the **FOIL method** reduces the rectangle method to a systematic approach without the rectangle. To develop the FOIL method, we use the distributive property to find $(x + 3)(x + 5)$.

$$\begin{aligned}(x + 3)(x + 5) &= (x + 3)x + (x + 3)5 \\ &= x(x) + 3(x) + x(5) + 3(5) \\ &= x^2 + 3x + 5x + 15 \\ &= x^2 + 8x + 15\end{aligned}$$

Here is where the letters of the word FOIL originate.

$(x + 3)(x + 5)$ Multiply the **First** terms: $x(x)$. **F**

$(x + 3)(x + 5)$ Multiply the **Outer** terms: $x(5)$. This is the **outer product.** **O**

$(x + 3)(x + 5)$ Multiply the **Inner** terms: $3(x)$. This is the **inner product.** **I**

$(x + 3)(x + 5)$ Multiply the **Last** terms: $3(5)$. **L**

The inner product and the outer product should be added mentally so that the three terms of the answer can be written without extra steps as

$$(x + 3)(x + 5) = x^2 + 8x + 15.$$

A summary of the steps in the FOIL method follows.

Multiplying Binomials by the FOIL Method

Step 1 Multiply the two **F**irst terms of the binomials to get the first term of the answer.

Step 2 Find the **O**uter product and the **I**nner product and add them (when possible) to get the middle term of the answer.

Step 3 Multiply the two **L**ast terms of the binomials to get the last term of the answer.

$\mathbf{F} = x^2 \quad \mathbf{L} = 15$

$(x + 3)(x + 5)$

I $3x$
O $5x$
$8x$ Add.

EXAMPLE 5 Using the FOIL Method

Use the FOIL method to find the product $(x + 8)(x - 6)$.

Step 1 **F** Multiply the first terms.

$$x(x) = x^2$$

Step 2 **O** Find the outer product.

$$x(-6) = -6x$$

I Find the inner product.

$$8(x) = 8x$$

Add the outer and inner products mentally.

$$-6x + 8x = 2x$$

Step 3 **L** Multiply the last terms.

$$8(-6) = -48$$

The product of $x + 8$ and $x - 6$ is the sum of the four terms found in three steps above, so

$$(x + 8)(x - 6) = x^2 + 2x - 48.$$

As a shortcut, this product can be found in the following manner.

First Last
$(x + 8)(x - 6)$
Inner
Outer

$x^2 \quad -48$
$(x + 8)(x - 6)$
$8x$
$-6x$
$2x$ Add.

Now Try Exercise 37.

It is not possible to add the inner and outer products of the FOIL method if unlike terms result, as shown in the next example.

EXAMPLE 6 Using the FOIL Method

Multiply $(9x - 2)(3y + 1)$.

First	$(9x - 2)(3y + 1)$	$27xy$	
Outer	$(9x - 2)(3y + 1)$	$9x$	Unlike terms
Inner	$(9x - 2)(3y + 1)$	$-6y$	
Last	$(9x - 2)(3y + 1)$	-2	

F O I L

$$(9x - 2)(3y + 1) = 27xy + 9x - 6y - 2$$

Now Try Exercise 51.

EXAMPLE 7 Using the FOIL Method

Find each product.

F O I L

(a) $(2k + 5y)(k + 3y) = 2k(k) + 2k(3y) + 5y(k) + 5y(3y)$

$= 2k^2 + 6ky + 5ky + 15y^2$

$= 2k^2 + 11ky + 15y^2$

(b) $(7p + 2q)(3p - q) = 21p^2 - pq - 2q^2$ FOIL

(c) $2x^2(x - 3)(3x + 4) = 2x^2(3x^2 - 5x - 12)$ FOIL

$= 6x^4 - 10x^3 - 24x^2$ Distributive property

Now Try Exercises 53 and 55.

NOTE Example 7(c) showed one way to multiply three polynomials. We could have multiplied $2x^2$ and $x - 3$ first, then multiplied that product and $3x + 4$ as follows.

$$2x^2(x - 3)(3x + 4) = (2x^3 - 6x^2)(3x + 4)$$
$$= 6x^4 - 10x^3 - 24x^2 \quad \text{FOIL}$$

5.5 EXERCISES

For Extra Help

Student's Solutions Manual
MyMathLab
InterAct Math Tutorial Software
AW Math Tutor Center
MathXL
Digital Video Tutor CD 10/Videotape 9

1. Match each product in Column I with the correct monomial in Column II.

I	II
(a) $(5x^3)(6x^5)$	**A.** $125x^{15}$
(b) $(-5x^5)(6x^3)$	**B.** $30x^8$
(c) $(5x^5)^3$	**C.** $-216x^9$
(d) $(-6x^3)^3$	**D.** $-30x^8$

2. Match each product in Column I with the correct polynomial in Column II.

I	II
(a) $(x - 5)(x + 3)$	**A.** $x^2 + 8x + 15$
(b) $(x + 5)(x + 3)$	**B.** $x^2 - 8x + 15$
(c) $(x - 5)(x - 3)$	**C.** $x^2 - 2x - 15$
(d) $(x + 5)(x - 3)$	**D.** $x^2 + 2x - 15$

Find each product. Use the rules for exponents discussed earlier in the chapter.

3. $(5y^4)(3y^7)$ **4.** $(10p^2)(5p^3)$ **5.** $(-15a^4)(-2a^5)$

6. $(-3m^6)(-5m^4)$ **7.** $(5p)(3q^2)$ **8.** $(4a^3)(3b^2)$

9. $(-6m^3)(3n^2)$ **10.** $(9r^3)(-2s^2)$

Find each product. See Example 1.

11. $2m(3m + 2)$ **12.** $4x(5x + 3)$

13. $3p(-2p^3 + 4p^2)$ **14.** $4x(3 + 2x + 5x^3)$

15. $-8z(2z + 3z^2 + 3z^3)$ **16.** $-7y(3 + 5y^2 - 2y^3)$

17. $2y^3(3 + 2y + 5y^4)$ **18.** $2m^4(3m^2 + 5m + 6)$

19. $-4r^3(-7r^2 + 8r - 9)$ **20.** $-9a^5(-3a^6 - 2a^4 + 8a^2)$

21. $3a^2(2a^2 - 4ab + 5b^2)$ **22.** $4z^3(8z^2 + 5zy - 3y^2)$

23. $7m^3n^2(3m^2 + 2mn - n^3)$ **24.** $2p^2q(3p^2q^2 - 5p + 2q^2)$

Find each product. See Examples 2–4.

25. $(6x + 1)(2x^2 + 4x + 1)$ **26.** $(9y - 2)(8y^2 - 6y + 1)$

27. $(9a + 2)(9a^2 + a + 1)$ **28.** $(2r - 1)(3r^2 + 4r - 4)$

29. $(4m + 3)(5m^3 - 4m^2 + m - 5)$ **30.** $(y + 4)(3y^4 - 2y^2 + 1)$

31. $(2x - 1)(3x^5 - 2x^3 + x^2 - 2x + 3)$ **32.** $(2a + 3)(a^4 - a^3 + a^2 - a + 1)$

33. $(5x^2 + 2x + 1)(x^2 - 3x + 5)$ **34.** $(2m^2 + m - 3)(m^2 - 4m + 5)$

35. $(6x^4 - 4x^2 + 8x)\left(\frac{1}{2}x + 3\right)$ **36.** $(8y^6 + 4y^4 - 12y^2)\left(\frac{3}{4}y^2 + 2\right)$

Find each product. Use the FOIL method. See Examples 5–7.

37. $(m + 7)(m + 5)$ **38.** $(n - 1)(n + 4)$ **39.** $(x + 5)(x - 5)$

40. $(y + 8)(y - 8)$ **41.** $(2x + 3)(6x - 4)$ **42.** $(4m + 3)(4m + 3)$

43. $(3x - 2)(3x - 2)$ **44.** $(b + 8)(6b - 2)$ **45.** $(5a + 1)(2a + 7)$

46. $(8 - 3a)(2 + a)$ **47.** $(6 - 5m)(2 + 3m)$ **48.** $(-4 + k)(2 - k)$

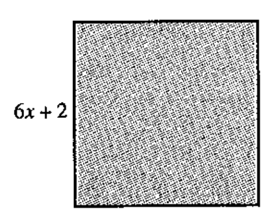
49. $(5 - 3x)(4 + x)$ **50.** $(2m - 3n)(m + 5n)$ **51.** $(4x + 3)(2y - 1)$

52. $(5x + 7)(3y - 8)$ **53.** $(3x + 2y)(5x - 3y)$

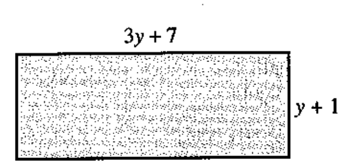
54. $x(2x - 5)(x + 3)$ **55.** $3y^3(2y + 3)(y - 5)$

56. $5t^4(t + 3)(3t - 1)$ **57.** $-8r^3(5r^2 + 2)(5r^2 - 2)$

58. Find a polynomial that represents the area of this square.



59. Find a polynomial that represents the area of this rectangle.



60. Perform the indicated multiplications, and then describe the pattern that you observe in the products.

(a) $(x + 4)(x - 4)$; $(y + 2)(y - 2)$; $(r + 7)(r - 7)$

(b) $(x + 4)(x + 4)$; $(y - 2)(y - 2)$; $(r + 7)(r + 7)$

Find each product. In Exercises 71–74 and 78–80, apply the meaning of exponents.

61. $\left(3p + \frac{5}{4}q\right)\left(2p - \frac{5}{3}q\right)$ **62.** $\left(-x + \frac{2}{3}y\right)\left(3x - \frac{3}{4}y\right)$

63. $(x + 7)^2$ **64.** $(m + 6)^2$

65. $(a - 4)(a + 4)$ **66.** $(b - 10)(b + 10)$

67. $(2p - 5)^2$ **68.** $(3m + 1)^2$

69. $(5k + 3q)^2$ **70.** $(8m - 3n)^2$

71. $(m - 5)^3$ **72.** $(p + 3)^3$

73. $(2a + 1)^3$ **74.** $(3m - 1)^3$

75. $7(4m - 3)(2m + 1)$ **76.** $-4r(3r + 2)(2r - 5)$

77. $-3a(3a + 1)(a - 4)$ **78.** $(k + 1)^4$

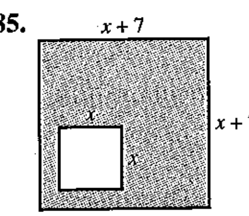
79. $(3r - 2s)^4$ **80.** $(2z + 5y)^4$

81. $3p^3(2p^2 + 5p)(p^3 + 2p + 1)$ **82.** $5k^2(k^2 - k + 4)(k^3 - 3)$

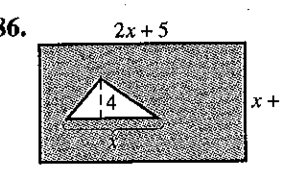
83. $-2x^5(3x^2 + 2x - 5)(4x + 2)$ **84.** $-4x^3(3x^4 + 2x^2 - x)(-2x + 1)$

Find a polynomial that represents the area of each shaded region. In Exercises 87 and 88 leave π in your answer. Use the formulas found on the inside covers.

85.



86.



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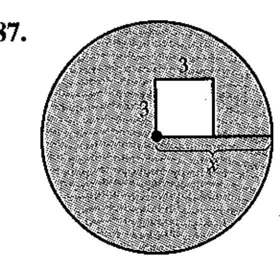
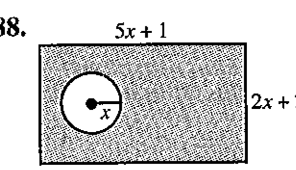


88.



RELATING CONCEPTS (EXERCISES 89–96)

For Individual or Group Work

Work Exercises 89–96 in order. *Refer to the figure as necessary.*

89. Find a polynomial that represents the area of the rectangle.

(continued)

90. Suppose you know that the area of the rectangle is 600 yd^2. Use this information and the polynomial from Exercise 89 to write an equation that allows you to solve for x.

91. Solve for x.

92. What are the dimensions of the rectangle (assume units are all in yards)?

93. Suppose the rectangle represents a lawn and it costs \$3.50 per yd^2 to lay sod on the lawn. How much will it cost to sod the entire lawn?

94. Use the result of Exercise 92 to find the perimeter of the lawn.

95. Again, suppose the rectangle represents a lawn and it costs \$9.00 per yd to fence the lawn. How much will it cost to fence the lawn?

96. **(a)** Suppose that it costs k dollars per yd^2 to sod the lawn. Determine a polynomial in the variables x and k that represents the cost to sod the entire lawn.

(b) Suppose that it costs r dollars per yd to fence the lawn. Determine a polynomial in the variables x and r that represents the cost to fence the lawn.

97. Explain the FOIL method for multiplying two binomials. Give an example.

98. Why does the FOIL method not apply to the product of a binomial and a trinomial? Give an example.

5.6 Special Products

OBJECTIVES

1. Square binomials.
2. Find the product of the sum and difference of two terms.
3. Find higher powers of binomials.

In this section, we develop shortcuts to find certain binomial products that occur frequently.

OBJECTIVE 1 Square binomials. The square of a binomial can be found quickly by using the method suggested by Example 1.

EXAMPLE 1 Squaring a Binomial

Find $(m + 3)^2$.

Squaring $m + 3$ by the FOIL method gives

$$(m + 3)(m + 3) = m^2 + 3m + 3m + 9$$
$$= m^2 + 6m + 9.$$

Now Try Exercise 1.

The result above has the squares of the first and the last terms of the binomial:

$$m^2 = m^2 \quad \text{and} \quad 3^2 = 9.$$

The middle term is twice the product of the two terms of the binomial, since the outer and inner products are $m(3)$ and $3(m)$, and

$$m(3) + 3(m) = 2(m)(3) = 6m.$$

This example suggests the following rules.