5.3 An Application of Exponents: Scientific Notation

OBJECTIVES

1. Express numbers in scientific notation.
2. Convert numbers in scientific notation to numbers without exponents.
3. Use scientific notation in calculations.

OBJECTIVE 1 Express numbers in scientific notation. One example of the use of exponents comes from science. The numbers occurring in science are often extremely large (such as the distance from Earth to the sun, 93,000,000 mi) or extremely small (the wavelength of yellow-green light, approximately .0000006 m). Because of the difficulty of working with many zeros, scientists often express such numbers with exponents. Each number is written as $a \times 10^n$, where $1 \leq |a| < 10$ and n is an integer. This form is called **scientific notation.** There is always one nonzero digit before the decimal point. This is shown in the following examples. (In work with scientific notation, the times symbol, $\times$, is commonly used.)

$3.19 \times 10^1 = 3.19 \times 10 = 31.9$	Decimal point moves 1 place to the right.
$3.19 \times 10^2 = 3.19 \times 100 = 319.$	Decimal point moves 2 places to the right.
$3.19 \times 10^3 = 3.19 \times 1000 = 3190.$	Decimal point moves 3 places to the right.
$3.19 \times 10^{-1} = 3.19 \times .1 = .319$	Decimal point moves 1 place to the left.
$3.19 \times 10^{-2} = 3.19 \times .01 = .0319$	Decimal point moves 2 places to the left.
$3.19 \times 10^{-3} = 3.19 \times .001 = .00319$	Decimal point moves 3 places to the left.

A number in scientific notation is always written with the decimal point after the first nonzero digit and then multiplied by the appropriate power of 10. For example, 35 is written 3.5×10^1, or 3.5×10; 56,200 is written 5.62×10^4, since

$$56{,}200 = 5.62 \times 10{,}000 = 5.62 \times 10^4.$$

To write a number in scientific notation, follow these steps.

Writing a Number in Scientific Notation

Step 1 Move the decimal point to the right of the first nonzero digit.

Step 2 Count the number of places you moved the decimal point.

Step 3 The number of places in Step 2 is the absolute value of the exponent on 10.

Step 4 The exponent on 10 is positive if the original number is larger than the number in Step 1; the exponent is negative if the original number is smaller than the number in Step 1. If the decimal point is not moved, the exponent is 0.

EXAMPLE 1 Using Scientific Notation

Write each number in scientific notation.

(a) 93,000,000

The number will be written in scientific notation as 9.3×10^n. To find the value of n, first compare the original number, 93,000,000, with 9.3. Here 93,000,000 is *larger* than 9.3. Therefore, multiply by a *positive* power of 10 so the product 9.3×10^n will equal the larger number.

Move the decimal point to follow the first nonzero digit (the 9). Count the number of places the decimal point was moved.

$$93{,}000{,}000 \quad \text{7 places}$$

Since the decimal point was moved 7 places, and since n is positive,

$$93{,}000{,}000 = 9.3 \times 10^7.$$

(b) $63{,}200{,}000{,}000 = 6.3200000000 = 6.32 \times 10^{10}$

10 places

(c) .00462

Move the decimal point to the right of the first nonzero digit, and count the number of places the decimal point was moved.

$$.00462 \quad \text{3 places}$$

Since .00462 is *smaller* than 4.62, the exponent must be *negative*.

$$.00462 = 4.62 \times 10^{-3}$$

(d) $.0000762 = 7.62 \times 10^{-5}$

Now Try Exercises 15 and 19.

NOTE To choose the exponent when writing a number in scientific notation, think: If the original number is "large," like 93,000,000, use a *positive* exponent on 10, since positive is larger than negative. However, if the original number is "small," like .00462, use a *negative* exponent on 10, since negative is smaller than positive.

OBJECTIVE 2 Convert numbers in scientific notation to numbers without exponents. To convert a number written in scientific notation to a number without exponents, work in reverse. Multiplying by a positive power of 10 will make the number larger; multiplying by a negative power of 10 will make the number smaller.

EXAMPLE 2 Writing Numbers without Exponents

Write each number without exponents.

(a) 6.2×10^3

Since the exponent is positive, make 6.2 larger by moving the decimal point 3 places to the right, inserting zeros as needed.

$$6.2 \times 10^3 = 6.200 = 6200$$

(b) $4.283 \times 10^5 = 4.28300 = 428{,}300$ Move 5 places to the right.

(c) $7.04 \times 10^{-3} = .00704$ Move 3 places to the left.

The exponent tells the number of places and the direction that the decimal point is moved.

Now Try Exercises 23 and 29.

OBJECTIVE 3 Use scientific notation in calculations. The next example uses scientific notation with products and quotients.

EXAMPLE 3 Multiplying and Dividing with Scientific Notation

Write each product or quotient without exponents.

(a) $(6 \times 10^3)(5 \times 10^{-4}) = (6 \times 5)(10^3 \times 10^{-4})$ — Commutative and associative properties

$= 30 \times 10^{-1}$ — Product rule

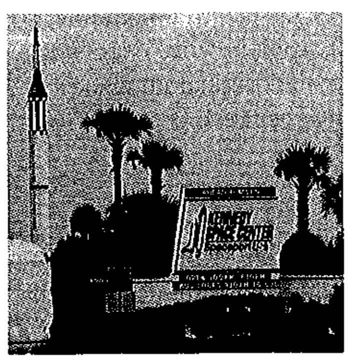
$= 3$ — Write without exponents.

(b) $\dfrac{4 \times 10^{-5}}{2 \times 10^3} = \dfrac{4}{2} \times \dfrac{10^{-5}}{10^3} = 2 \times 10^{-8} = .00000002$

Now Try Exercises 33 and 43.

NOTE Multiplying or dividing numbers written in scientific notation may produce an answer in the form $a \times 10^0$. Since $10^0 = 1$, $a \times 10^0 = a$. For example,

$$(8 \times 10^{-4})(5 \times 10^4) = 40 \times 10^0 = 40.$$



EXAMPLE 4 Calculating Using Scientific Notation

Convert to scientific notation, perform each computation, then give the result without scientific notation.

(a) To determine helium usage at Kennedy Space Center, scientists must calculate 70,000(.0283)(1000). (*Source: NASA-AMATYC-NSF Mathematics Explorations II,* Capital Community College, 2000.)

$$\begin{aligned} 70{,}000(.0283)(1000) &= (7 \times 10^4)(2.83 \times 10^{-2})(1 \times 10^3) \\ &= (7 \times 2.83 \times 1)(10^{4-2+3}) \\ &= 19.81 \times 10^5 \\ &= 1{,}981{,}000 \end{aligned}$$

(b) The ratio of the tidal force exerted by the moon compared to that exerted by the sun is given by

$$\frac{73.5 \times 10^{21} \times (1.5 \times 10^8)^3}{1.99 \times 10^{30} \times (3.84 \times 10^5)^3}.$$

(*Source:* Kastner, Bernice, *Space Mathematics,* NASA.)

$$\begin{aligned} \frac{7.35 \times 10^1 \times 10^{21} \times 1.5^3 \times 10^{24}}{1.99 \times 10^{30} \times 3.84^3 \times 10^{15}} &\approx .22 \times 10^{1+21+24-30-15} \\ &= .22 \times 10^1 \\ &= 2.2 \end{aligned}$$

Now Try Exercises 61 and 65.

CONNECTIONS

Charles F. Richter devised a scale in 1935 to compare the intensities, or relative power, of earthquakes. The *intensity* of an earthquake is measured relative to the intensity of a standard *zero-level* earthquake of intensity I_0. The relationship is equivalent to $I = I_0 \times 10^R$, where R is the *Richter scale* measure. For example, if an earthquake has magnitude 5.0 on the Richter scale, then its intensity is calculated as $I = I_0 \times 10^{5.0} = I_0 \times 100{,}000$, which is 100,000 times as intense as a zero-level earthquake. The following diagram illustrates the intensities of earthquakes and their Richter scale magnitudes.

Intensity	$I_0 \times 10^0$	$I_0 \times 10^1$	$I_0 \times 10^2$	$I_0 \times 10^3$	$I_0 \times 10^4$	$I_0 \times 10^5$	$I_0 \times 10^6$	$I_0 \times 10^7$	$I_0 \times 10^8$
Richter Scale	0	1	2	3	4	5	6	7	8

To compare two earthquakes to each other, a ratio of the intensities is calculated. For example, to compare an earthquake that measures 8.0 on the Richter scale to one that measures 5.0, simply find the ratio of the intensities:

$$\frac{\text{intensity } 8.0}{\text{intensity } 5.0} = \frac{I_0 \times 10^{8.0}}{I_0 \times 10^{5.0}} = \frac{10^8}{10^5} = 10^{8-5} = 10^3 = 1000.$$

Therefore an earthquake that measures 8.0 on the Richter Scale is 1000 times as intense as one that measures 5.0.

For Discussion or Writing

The table gives Richter scale measurements for several earthquakes.

Earthquake	Richter Scale Measurement
1960 Concepción, Chile	9.5
1906 San Francisco, California	8.3
1939 Erzincan, Turkey	8.0
1998 Sumatra, Indonesia	7.0
1998 Adana, Turkey	6.3

Source: World Almanac and Books of Facts, 2000.

1. Compare the intensity of the 1939 Erzincan earthquake to the 1998 Sumatra earthquake.
2. Compare the intensity of the 1998 Adana earthquake to the 1906 San Francisco earthquake.
3. Compare the intensity of the 1939 Erzincan earthquake to the 1998 Adana earthquake.
4. Suppose an earthquake measures 7.2 on the Richter scale. How would the intensity of a second earthquake compare if its Richter scale measure differed by +3.0? By −1.0?

5.3 EXERCISES

For Extra Help

- Student's Solutions Manual
- MyMathLab
- InterAct Math Tutorial Software
- AW Math Tutor Center
- MathXL
- Digital Video Tutor CD 9/Videotape 8

Match each number written in scientific notation in Column I with the correct choice from Column II.

I	II
1. 4.6×10^{-4}	**A.** .00046
2. 4.6×10^{4}	**B.** 46,000
3. 4.6×10^{5}	**C.** 460,000
4. 4.6×10^{-5}	**D.** .000046

Determine whether or not each number is written in scientific notation as defined in Objective 1. If it is not, write it as such.

5. 4.56×10^{3} **6.** 7.34×10^{5} **7.** 5,600,000 **8.** 34,000

9. $.8 \times 10^{2}$ **10.** $.9 \times 10^{3}$ **11.** .004 **12.** .0007

13. Explain in your own words what it means for a number to be written in scientific notation. Give examples.

14. Explain how to multiply a number by a positive power of ten. Then explain how to multiply a number by a negative power of ten.

Write each number in scientific notation. See Example 1.

15. 5,876,000,000 **16.** 9,994,000,000 **17.** 82,350 **18.** 78,330

19. .000007 **20.** .0000004 **21.** .00203 **22.** .0000578

Write each number without exponents. See Example 2.

23. 7.5×10^{5} **24.** 8.8×10^{6} **25.** 5.677×10^{12} **26.** 8.766×10^{9}

27. 6.21×10^{0} **28.** 8.56×10^{0} **29.** 7.8×10^{-4} **30.** 8.9×10^{-5}

31. 5.134×10^{-9} **32.** 7.123×10^{-10}

Use properties and rules for exponents to perform the indicated operations, and write each answer without exponents. See Example 3.

33. $(2 \times 10^{8}) \times (3 \times 10^{3})$ **34.** $(4 \times 10^{7}) \times (3 \times 10^{3})$

35. $(5 \times 10^{4}) \times (3 \times 10^{2})$ **36.** $(8 \times 10^{5}) \times (2 \times 10^{3})$

37. $(3 \times 10^{-4}) \times (2 \times 10^{8})$ **38.** $(4 \times 10^{-3}) \times (2 \times 10^{7})$

39. $\dfrac{9 \times 10^{-5}}{3 \times 10^{-1}}$ **40.** $\dfrac{12 \times 10^{-4}}{4 \times 10^{-3}}$ **41.** $\dfrac{8 \times 10^{3}}{2 \times 10^{2}}$

42. $\dfrac{5 \times 10^{4}}{1 \times 10^{3}}$ **43.** $\dfrac{2.6 \times 10^{-3}}{2 \times 10^{2}}$ **44.** $\dfrac{9.5 \times 10^{-1}}{5 \times 10^{3}}$

TECHNOLOGY INSIGHTS (EXERCISES 45–50)

Graphing calculators such as the TI-83 Plus can display numbers in scientific notation (when in scientific mode), using the format shown in the screen at the top left on the next page. For example, the calculator displays 5.4E3 *to represent* 5.4×10^{3}, *the scientific notation form for* 5400. *The display* 5.4E−4 *means* 5.4×10^{-4}. *It will also perform operations with numbers entered in scientific notation, as shown in the screen on the right. Notice how the rules for exponents are applied.*


```
5400
            5.4E3
5.4*10^(-4)
           5.4E-4
```

```
(2E5)*(4E-2)
              8E3
(2E5)/(4E-2)
              5E6
(2E5)³
             8E15
```

Predict the display the calculator would give for the expression shown in each screen.

45. `.00000047`

46. `.000021`

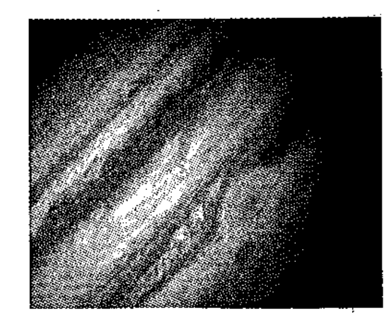
47. `(8E5)/(4E-2)`

48. `(9E-4)/(3E3)`

49. `(2E6)*(2E-3)/(4E2)`

50. `(5E-3)*(1E9)/(5E3)`

Each statement comes from Astronomy! A Brief Edition *by James B. Kaler (Addison-Wesley, 1997). If the number in italics is in scientific notation, write it without exponents. If the number is written without exponents, write it in scientific notation.*



51. Multiplying this view over the whole sky yields a galaxy count of more than *10 billion.* (page 496)

52. The circumference of the solar orbit is . . . about *4.7 million* km . . . . (in reference to the orbit of Jupiter, page 395)

53. The solar luminosity requires that 2×10^{9} kg of mass be converted into energy every second. (page 327)

54. At maximum, a cosmic ray particle—a mere atomic nucleus of only 10^{-13} cm across—can carry the energy of a professionally pitched baseball. (page 445)

Each of the following statements contains a number in italics. Write the number in scientific notation.

55. During the 1999–2000 Broadway season, gross receipts were *$603,000,000.* (*Source:* The League of American Theatres and Producers, Inc.)

56. In 1999, the leading U.S. advertiser was the General Motors Corporation, which spent approximately *$2,900,000,000.* (*Source:* Competitive Media Reporting and Publishers Information Bureau.)

57. In 2000, service revenue of the cellular telephone industry was *$52,466,000,000.* (*Source:* Cellular Telecommunications & Internet Association.)

58. Assets of the insured commercial banks in the state of New York totaled *$1,304,300,000.* (*Source:* U.S. Federal Deposit Insurance Corporation.)

Use scientific notation to calculate the answer to each problem. See Example 4.

59. The distance to Earth from the planet Pluto is 4.58×10^9 km. In April 1983, Pioneer 10 transmitted radio signals from Pluto to Earth at the speed of light, 3.00×10^5 km per sec. How long (in seconds) did it take for the signals to reach Earth?

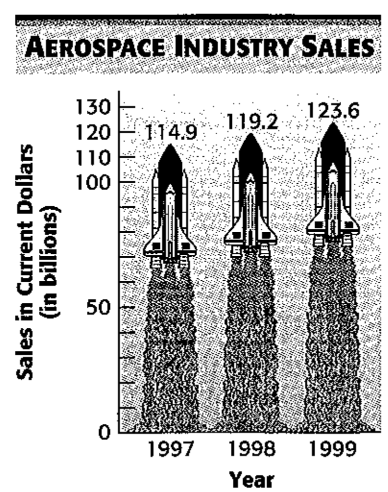
60. In Exercise 59, how many hours did it take for the signals to reach Earth?

61. In a recent year, the state of Texas had about 1.3×10^6 farms with an average of 7.1×10^2 acres per farm. What was the total number of acres devoted to farmland in Texas that year? (*Source:* National Agricultural Statistics Service, U.S. Department of Agriculture.)

62. The graph depicts aerospace industry sales. The figures at the tops of the bars represent billions of dollars.

(a) For each year, write the figure in scientific notation.

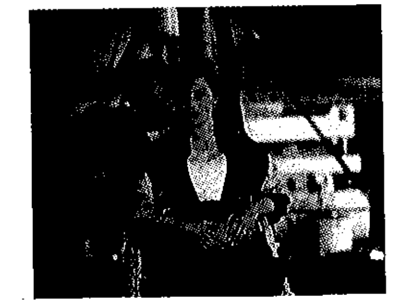
(b) If a line segment is drawn between the tops of the bars for 1997 and 1999, what is its slope?



Source: U.S. Bureau of the Census, *Current Industrial Reports.*

63. There are 10^9 Social Security numbers. The population of the United States is about 3×10^8. How many Social Security numbers are available for each person? (*Source:* U.S. Bureau of the Census.)

64. The top-grossing movie of 1997 was *Titanic,* with box office receipts of about 6×10^8 dollars. That amount represented a fraction of about 9.5×10^{-3} of the total receipts for motion pictures in that year. What were the total receipts? (*Source:* U.S. Bureau of the Census.)



65. The body of a 150-lb person contains about 2.3×10^{-4} lb of copper. How much copper is contained in the bodies of 1200 such people?

66. There were 6.3×10^{10} dollars spent to attend motion pictures in a recent year. Approximately 1.3×10^8 adults attended a motion picture theater at least once. What was the average amount spent per person that year? (*Source:* U.S. National Endowment for the Arts.)

5.4 Adding and Subtracting Polynomials; Graphing Simple Polynomials

OBJECTIVES

1. Identify terms and coefficients.
2. Add like terms.
3. Know the vocabulary for polynomials.
4. Evaluate polynomials.
5. Add and subtract polynomials.
6. Graph equations defined by polynomials of degree 2.

OBJECTIVE 1 Identify terms and coefficients. In Chapter 1 we saw that in an expression such as

$$4x^3 + 6x^2 + 5x + 8,$$

the quantities $4x^3$, $6x^2$, $5x$, and 8 are called *terms.* As mentioned earlier, in the term $4x^3$, the number 4 is called the *numerical coefficient,* or simply the *coefficient,* of x^3. In the same way, 6 is the coefficient of x^2 in the term $6x^2$, 5 is the coefficient of x in the term $5x$, and 8 is the coefficient in the term 8. A constant term, like 8 in the expression above, can be thought of as $8 \cdot 1 = 8x^0$, since $x^0 = 1$.

EXAMPLE 1 Identifying Coefficients

Name the (numerical) coefficient of each term in these expressions.

(a) $4x^3$

The coefficient is 4.

(b) $x - 6x^4$

The coefficient of x is 1 because $x = 1 \cdot x$ or $1x$. The coefficient of x^4 is -6 since we can write $x - 6x^4$ as the sum $x + (-6x^4)$.

(c) $5 - v^3$

The coefficient of the term 5 is 5 because $5 = 5v^0$. By writing $5 - v^3$ as a sum, $5 + (-v^3)$, or $5 + (-1v^3)$, we can identify the coefficient of v^3 as -1.

Now Try Exercises 9 and 13.

OBJECTIVE 2 Add like terms. Recall from Section 1.8 that *like terms* have exactly the same combination of variables with the same exponents on the variables. Only